#6

Application Serial No.: 10/042,883
Title: METHOD AND APPARATUS FOR SOLVING AN INEQUALITY CONSTRAINED GLOBAL OPTIMIZATION PROBLEM
Inventor(s): G. William Walster et al.

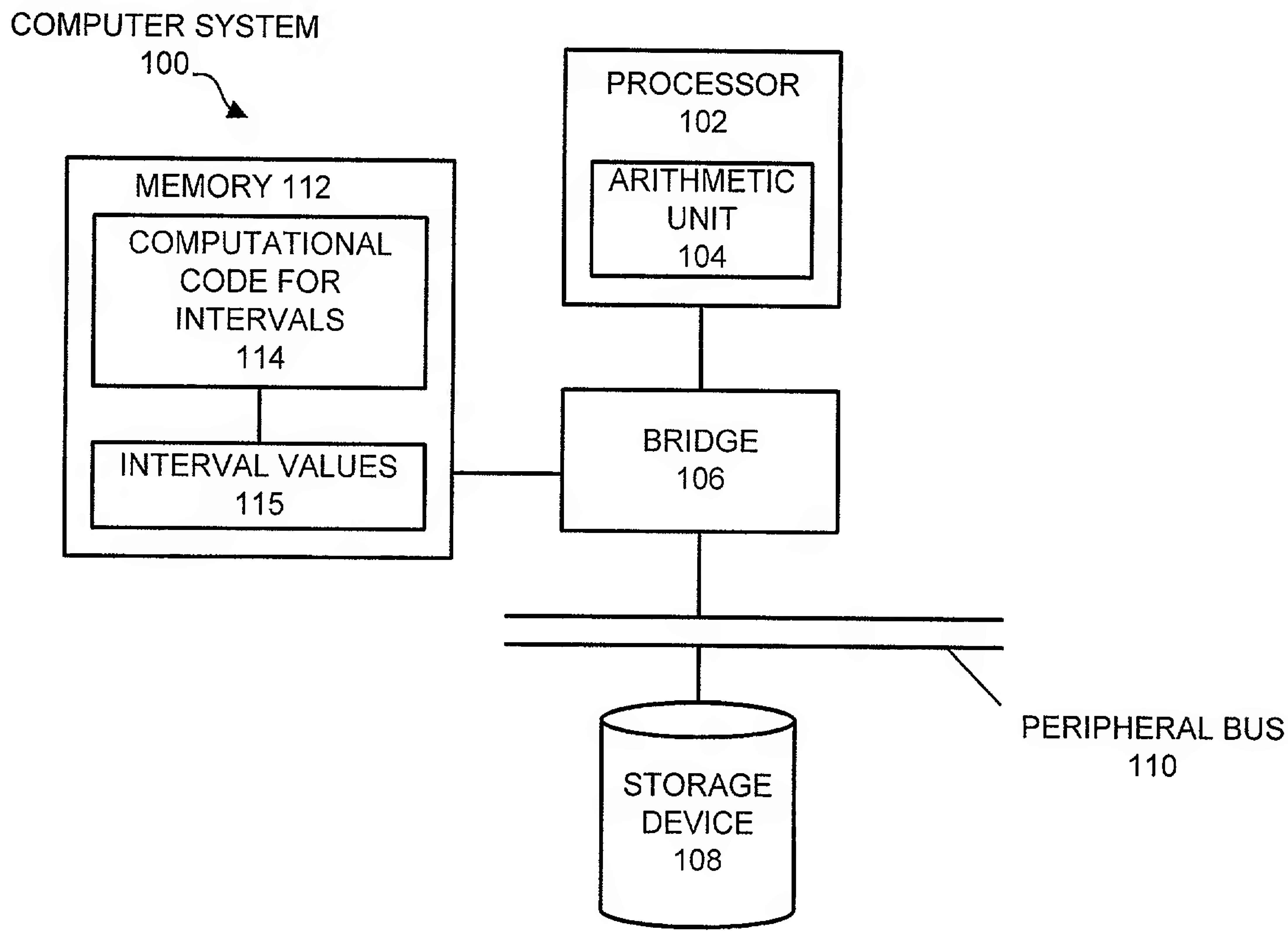

FIG. 1

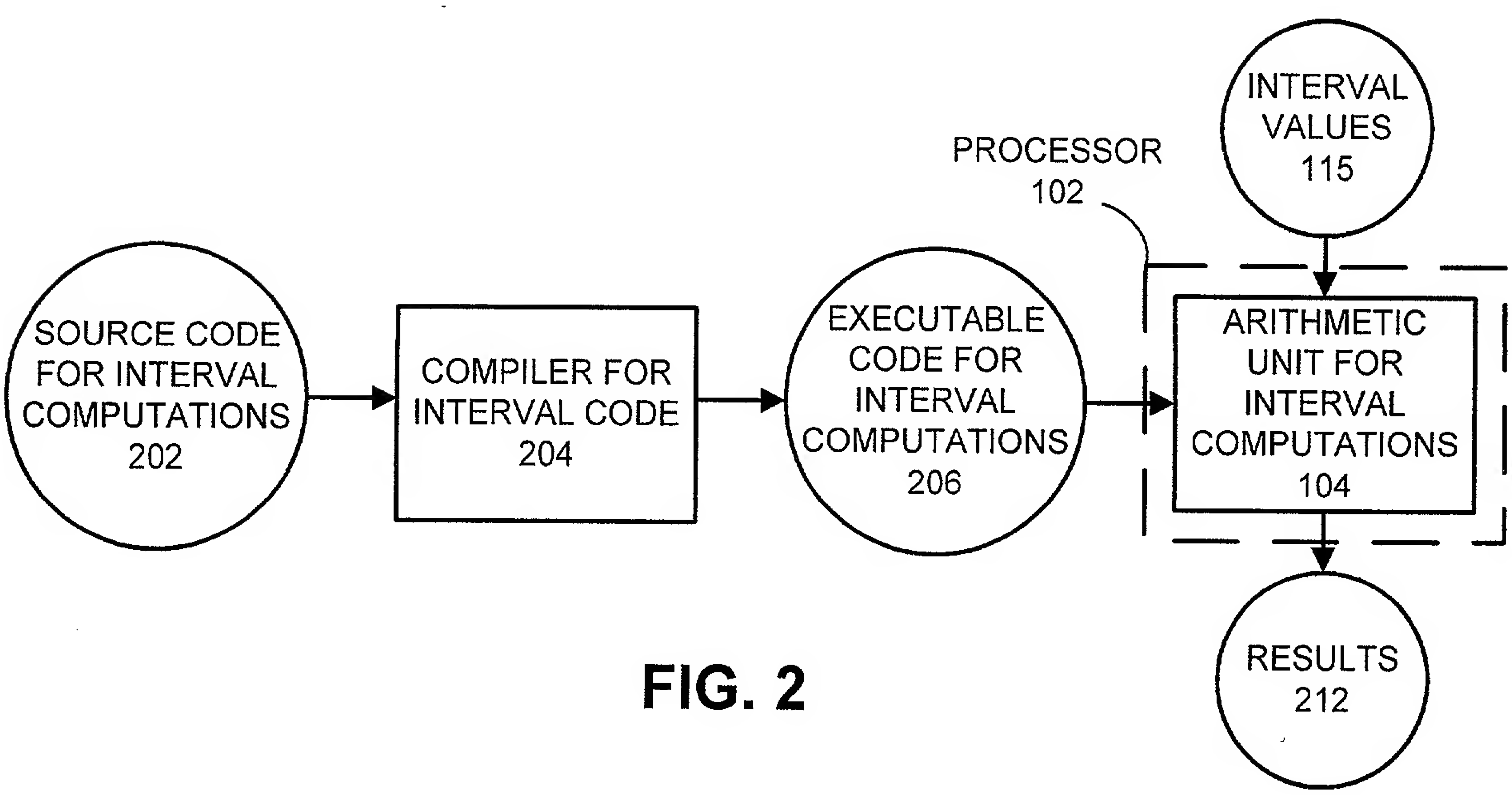

FIG. 2

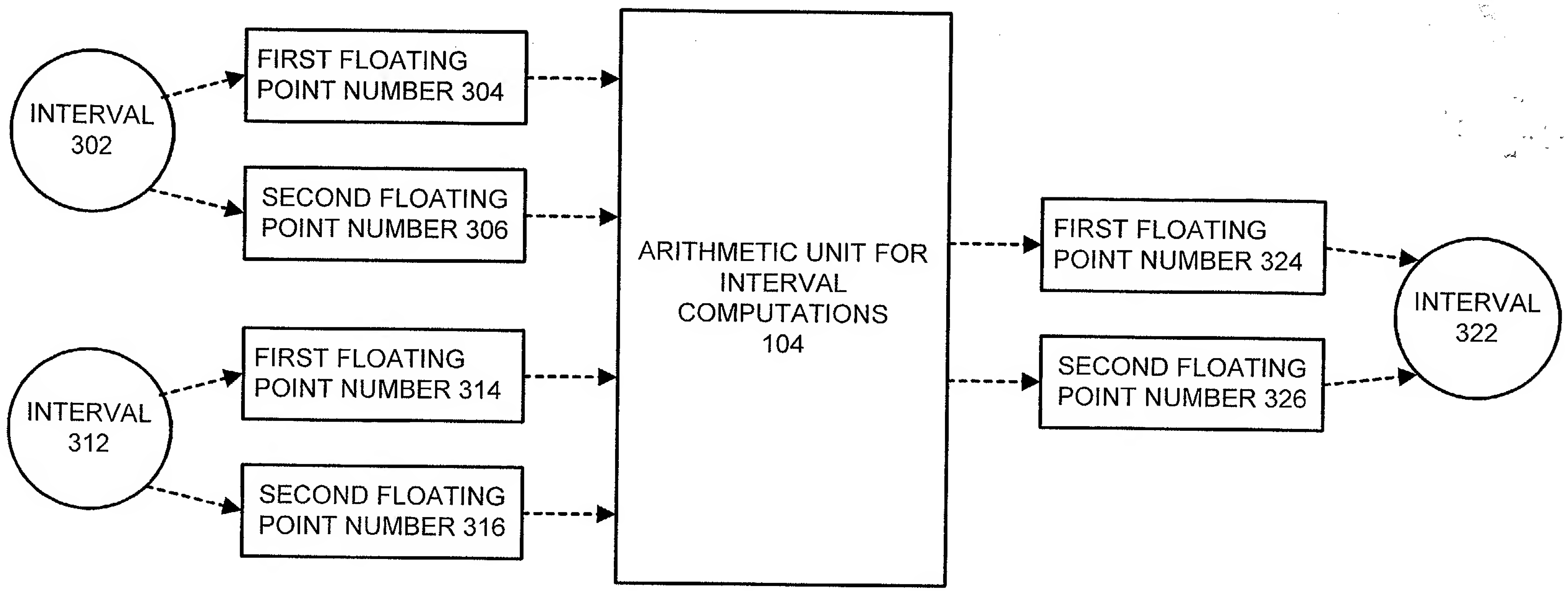
INTERVAL
302
FIRST FLOATING
POINT NUMBER 304
SECOND FLOATING
POINT NUMBER 306
INTERVAL
312
FIRST FLOATING
POINT NUMBER 314
SECOND FLOATING
POINT NUMBER 316
ARITHMETIC UNIT FOR
INTERVAL
COMPUTATIONS
104
FIRST FLOATING
POINT NUMBER 324
SECOND FLOATING
POINT NUMBER 326
INTERVAL
322

FIG. 3

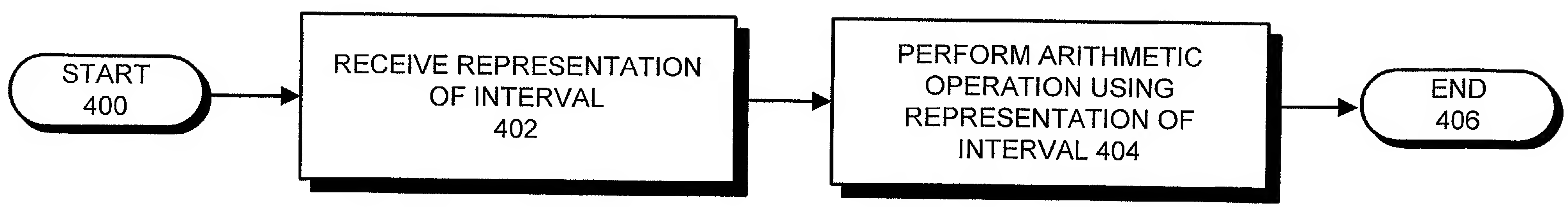
START
400
RECEIVE REPRESENTATION
OF INTERVAL
402
PERFORM ARITHMETIC
OPERATION USING
REPRESENTATION OF
INTERVAL 404
END
406

FIG. 4

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$$X \equiv [\underline{x}, \overline{x}] \equiv \{x \in \Re^* \mid \underline{x} \le x \le \overline{x}\}$$

$$Y \equiv [\underline{y}, \overline{y}] \equiv \{y \in \Re^* \mid \underline{y} \le y \le \overline{y}\}$$

$$(1)\quad X + Y = [\downarrow \underline{x} + \underline{y}, \uparrow \overline{x} + \overline{y}]$$

$$(2)\quad X - Y = [\downarrow \underline{x} - \overline{y}, \uparrow \overline{x} - \underline{y}]$$

$$(3)\quad X \times Y = [\min(\downarrow \underline{x} \times \underline{y}, \underline{x} \times \overline{y}, \overline{x} \times \underline{y}, \overline{x} \times \overline{y}), \max(\uparrow \underline{x} \times \underline{y}, \underline{x} \times \overline{y}, \overline{x} \times \underline{y}, \overline{x} \times \overline{y})]$$

$$(4)\quad X / Y = [\min(\downarrow \underline{x} / \underline{y}, \underline{x} / \overline{y}, \overline{x} / \underline{y}, \overline{x} / \overline{y}), \max(\uparrow \underline{x} / \underline{y}, \underline{x} / \overline{y}, \overline{x} / \underline{y}, \overline{x} / \overline{y})], \text{ if } 0 \notin Y$$

$$X / Y \subseteq \Re^*, \text{ if } 0 \in Y$$

FIG. 5

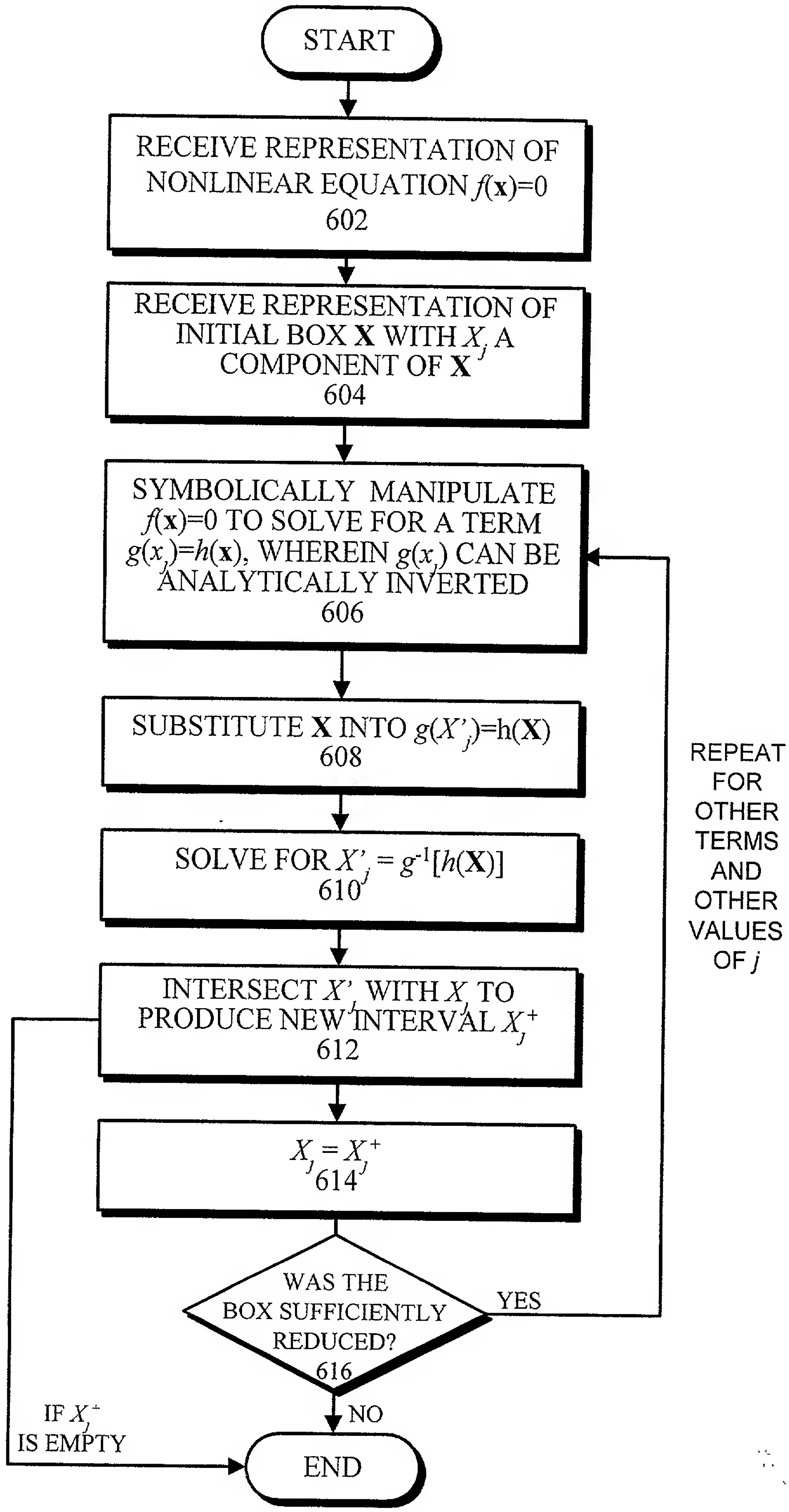
START
RECEIVE REPRESENTATION OF NONLINEAR EQUATION $f(\mathbf{x})=0$
602
RECEIVE REPRESENTATION OF INITIAL BOX $\mathbf{X}$ WITH X_j A COMPONENT OF $\mathbf{X}$
604
SYMBOLICALLY MANIPULATE $f(\mathbf{x})=0$ TO SOLVE FOR A TERM $g(x_j)=h(\mathbf{x})$, WHEREIN $g(x_j)$ CAN BE ANALYTICALLY INVERTED
606
SUBSTITUTE $\mathbf{X}$ INTO $g(X'_j)=h(\mathbf{X})$
608
SOLVE FOR $X'_j = g^{-1}[h(\mathbf{X})]$
610
INTERSECT X'_j WITH X_j TO PRODUCE NEW INTERVAL X_j^+
612
$X_j = X_j^+$
614
WAS THE BOX SUFFICIENTLY REDUCED?
616
YES
NO
REPEAT FOR OTHER TERMS AND OTHER VALUES OF j
IF X_j^+ IS EMPTY
END

FIG. 6

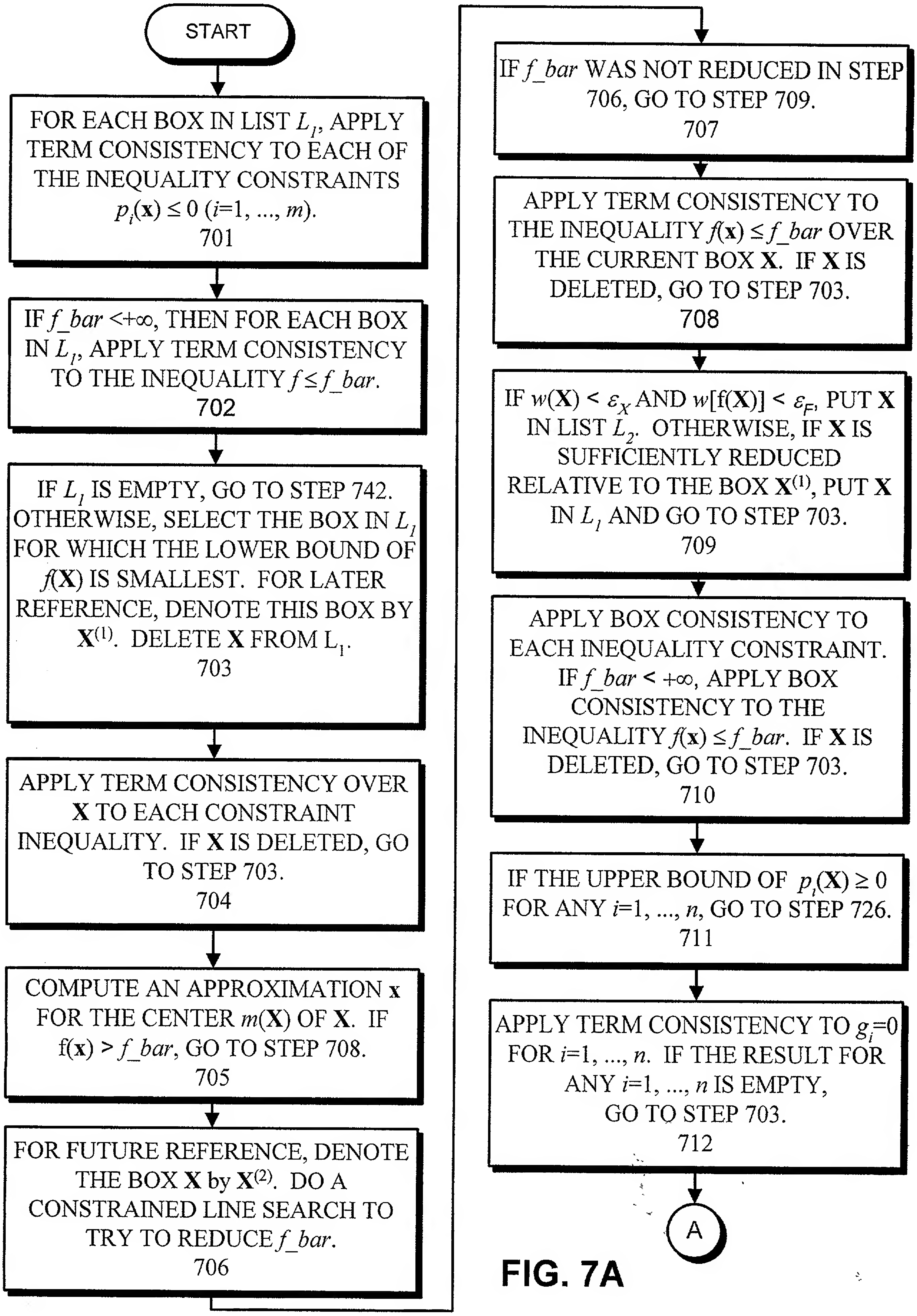
START
FOR EACH BOX IN LIST L_1, APPLY TERM CONSISTENCY TO EACH OF THE INEQUALITY CONSTRAINTS $p_i(\mathbf{x}) \leq 0$ (i=1, ..., m). 701
IF f_bar <+∞, THEN FOR EACH BOX IN L_1, APPLY TERM CONSISTENCY TO THE INEQUALITY $f \leq f\_bar$. 702
IF L_1 IS EMPTY, GO TO STEP 742. OTHERWISE, SELECT THE BOX IN L_1 FOR WHICH THE LOWER BOUND OF $f(\mathbf{X})$ IS SMALLEST. FOR LATER REFERENCE, DENOTE THIS BOX BY $\mathbf{X}^{(1)}$. DELETE $\mathbf{X}$ FROM L_1. 703
APPLY TERM CONSISTENCY OVER $\mathbf{X}$ TO EACH CONSTRAINT INEQUALITY. IF $\mathbf{X}$ IS DELETED, GO TO STEP 703. 704
COMPUTE AN APPROXIMATION $\mathbf{x}$ FOR THE CENTER $m(\mathbf{X})$ OF $\mathbf{X}$. IF $f(\mathbf{x}) > f\_bar$, GO TO STEP 708. 705
FOR FUTURE REFERENCE, DENOTE THE BOX $\mathbf{X}$ by $\mathbf{X}^{(2)}$. DO A CONSTRAINED LINE SEARCH TO TRY TO REDUCE f_bar. 706
IF f_bar WAS NOT REDUCED IN STEP 706, GO TO STEP 709. 707
APPLY TERM CONSISTENCY TO THE INEQUALITY $f(\mathbf{x}) \leq f\_bar$ OVER THE CURRENT BOX $\mathbf{X}$. IF $\mathbf{X}$ IS DELETED, GO TO STEP 703. 708
IF $w(\mathbf{X}) < \varepsilon_X$ AND $w[f(\mathbf{X})] < \varepsilon_F$, PUT $\mathbf{X}$ IN LIST L_2. OTHERWISE, IF $\mathbf{X}$ IS SUFFICIENTLY REDUCED RELATIVE TO THE BOX $\mathbf{X}^{(1)}$, PUT $\mathbf{X}$ IN L_1 AND GO TO STEP 703. 709
APPLY BOX CONSISTENCY TO EACH INEQUALITY CONSTRAINT. IF f_bar < +∞, APPLY BOX CONSISTENCY TO THE INEQUALITY $f(\mathbf{x}) \leq f\_bar$. IF $\mathbf{X}$ IS DELETED, GO TO STEP 703. 710
IF THE UPPER BOUND OF $p_i(\mathbf{X}) \geq 0$ FOR ANY i=1, ..., n, GO TO STEP 726. 711
APPLY TERM CONSISTENCY TO g_i=0 FOR i=1, ..., n. IF THE RESULT FOR ANY i=1, ..., n IS EMPTY, GO TO STEP 703. 712
A

FIG. 7A

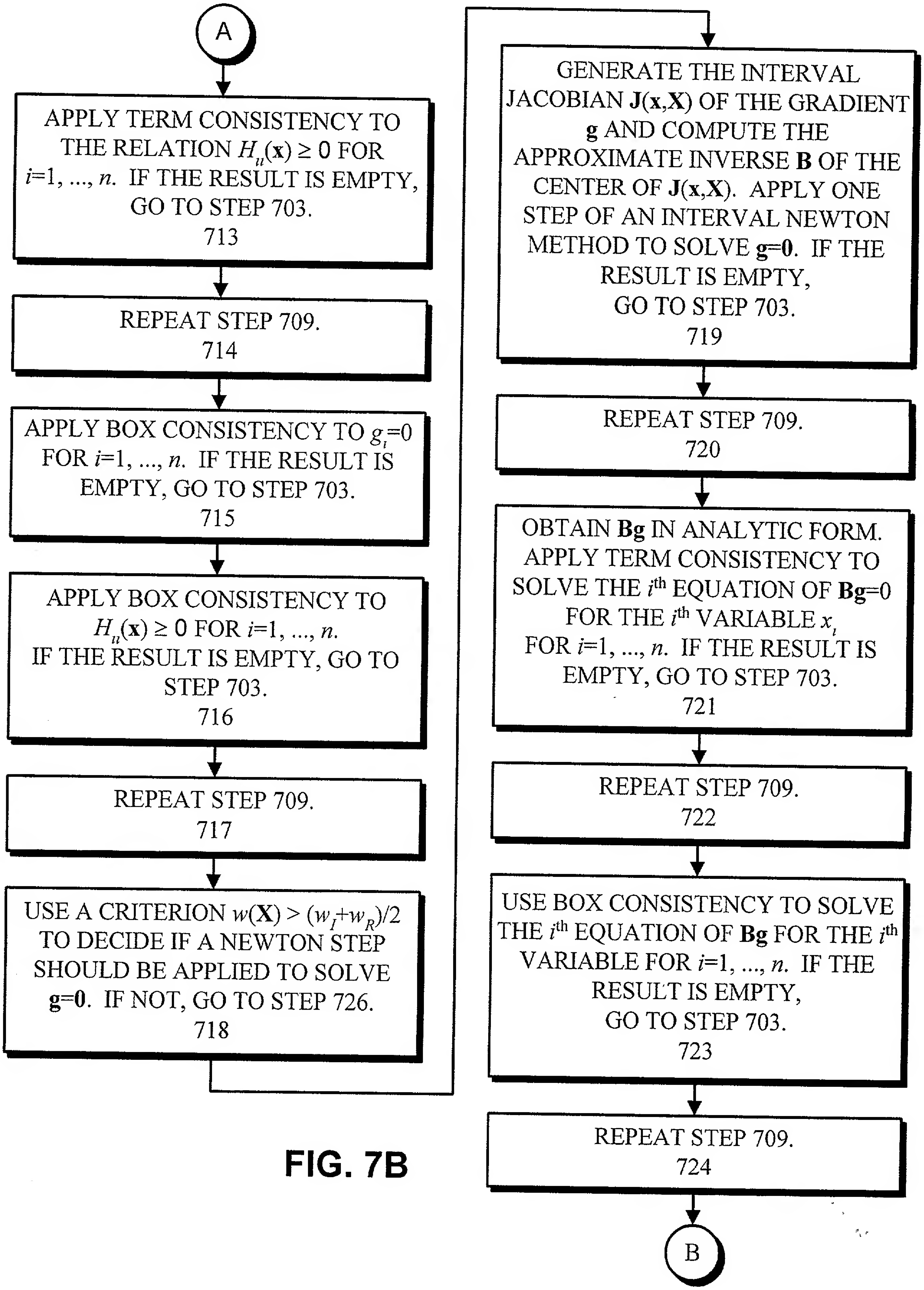
A
APPLY TERM CONSISTENCY TO THE RELATION $H_u(\mathbf{x}) \geq 0$ FOR i=1, ..., n. IF THE RESULT IS EMPTY, GO TO STEP 703.
713
REPEAT STEP 709.
714
APPLY BOX CONSISTENCY TO g_i=0 FOR i=1, ..., n. IF THE RESULT IS EMPTY, GO TO STEP 703.
715
APPLY BOX CONSISTENCY TO $H_u(\mathbf{x}) \geq 0$ FOR i=1, ..., n. IF THE RESULT IS EMPTY, GO TO STEP 703.
716
REPEAT STEP 709.
717
USE A CRITERION $w(\mathbf{X}) > (w_I+w_R)/2$ TO DECIDE IF A NEWTON STEP SHOULD BE APPLIED TO SOLVE **g**=**0**. IF NOT, GO TO STEP 726.
718
GENERATE THE INTERVAL JACOBIAN **J**(**x**,**X**) OF THE GRADIENT **g** AND COMPUTE THE APPROXIMATE INVERSE **B** OF THE CENTER OF **J**(**x**,**X**). APPLY ONE STEP OF AN INTERVAL NEWTON METHOD TO SOLVE **g**=**0**. IF THE RESULT IS EMPTY, GO TO STEP 703.
719
REPEAT STEP 709.
720
OBTAIN **Bg** IN ANALYTIC FORM. APPLY TERM CONSISTENCY TO SOLVE THE i^{th} EQUATION OF **Bg**=0 FOR THE i^{th} VARIABLE x_i FOR i=1, ..., n. IF THE RESULT IS EMPTY, GO TO STEP 703.
721
REPEAT STEP 709.
722
USE BOX CONSISTENCY TO SOLVE THE i^{th} EQUATION OF **Bg** FOR THE i^{th} VARIABLE FOR i=1, ..., n. IF THE RESULT IS EMPTY, GO TO STEP 703.
723
REPEAT STEP 709.
724
B

FIG. 7B

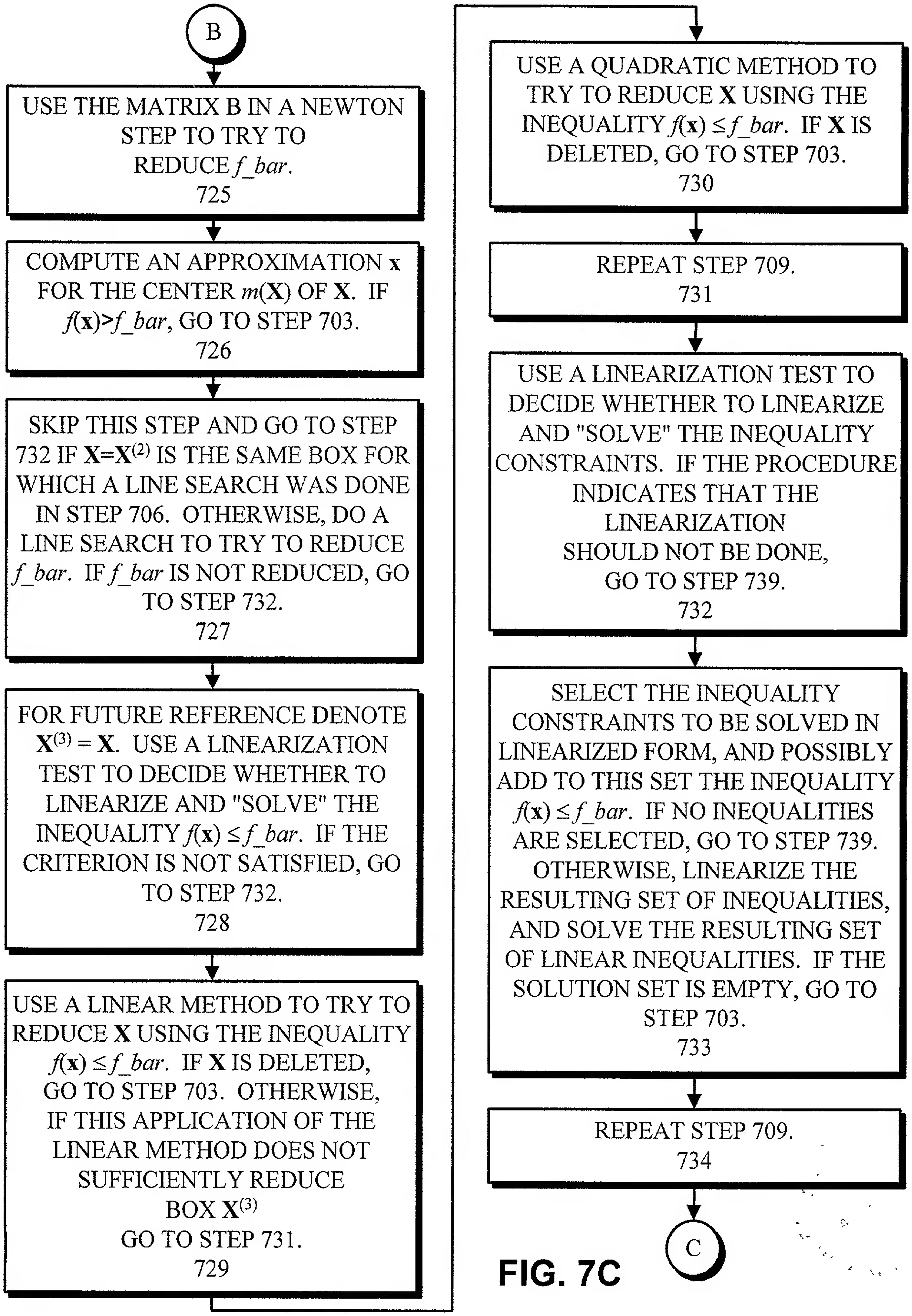
B
USE THE MATRIX B IN A NEWTON STEP TO TRY TO REDUCE f_bar.
725
COMPUTE AN APPROXIMATION x FOR THE CENTER m(X) OF X. IF f(x)>f_bar, GO TO STEP 703.
726
SKIP THIS STEP AND GO TO STEP 732 IF X=X(2) IS THE SAME BOX FOR WHICH A LINE SEARCH WAS DONE IN STEP 706. OTHERWISE, DO A LINE SEARCH TO TRY TO REDUCE f_bar. IF f_bar IS NOT REDUCED, GO TO STEP 732.
727
FOR FUTURE REFERENCE DENOTE X(3) = X. USE A LINEARIZATION TEST TO DECIDE WHETHER TO LINEARIZE AND "SOLVE" THE INEQUALITY f(x) ≤ f_bar. IF THE CRITERION IS NOT SATISFIED, GO TO STEP 732.
728
USE A LINEAR METHOD TO TRY TO REDUCE X USING THE INEQUALITY f(x) ≤ f_bar. IF X IS DELETED, GO TO STEP 703. OTHERWISE, IF THIS APPLICATION OF THE LINEAR METHOD DOES NOT SUFFICIENTLY REDUCE BOX X(3) GO TO STEP 731.
729
USE A QUADRATIC METHOD TO TRY TO REDUCE X USING THE INEQUALITY f(x) ≤ f_bar. IF X IS DELETED, GO TO STEP 703.
730
REPEAT STEP 709.
731
USE A LINEARIZATION TEST TO DECIDE WHETHER TO LINEARIZE AND "SOLVE" THE INEQUALITY CONSTRAINTS. IF THE PROCEDURE INDICATES THAT THE LINEARIZATION SHOULD NOT BE DONE, GO TO STEP 739.
732
SELECT THE INEQUALITY CONSTRAINTS TO BE SOLVED IN LINEARIZED FORM, AND POSSIBLY ADD TO THIS SET THE INEQUALITY f(x) ≤ f_bar. IF NO INEQUALITIES ARE SELECTED, GO TO STEP 739. OTHERWISE, LINEARIZE THE RESULTING SET OF INEQUALITIES, AND SOLVE THE RESULTING SET OF LINEAR INEQUALITIES. IF THE SOLUTION SET IS EMPTY, GO TO STEP 703.
733
REPEAT STEP 709.
734
C

FIG. 7C

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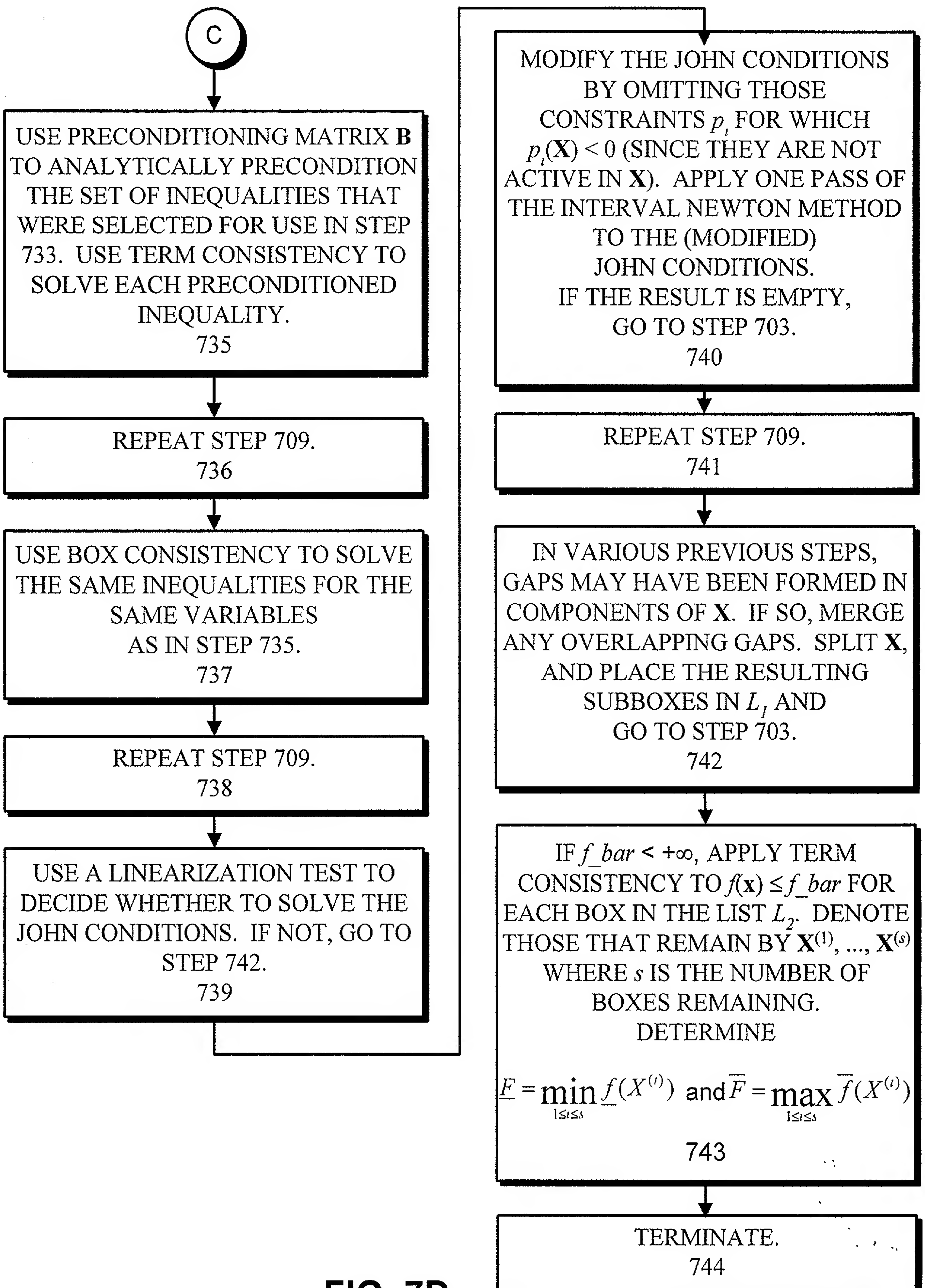

FIG. 7D